Секция: экономическая теория

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STRUCTURAL ESTIMATION OF KEYWORD AUCTION

This paper considers a model of real-time keyword auctions such as those used by Yandex, Google, Yahoo! and the other major search engines to sell sponsored-link positions. Our main goal is to estimate convergence properties of the two most commonly used in practice types of mechanism – the GSP and the VCG – and compare the auctioneer revenue obtainable with their help. We examine a several greedy bidding strategies and represent the most interesting case among such dynamic strategies. We will also answer the question whether we should expect this strategy to converge to a Nash equilibrium, and explore this problem for two possible situations: synchronous and asynchronous models in the GSP mechanism.

The main problems of Internet advertising - which advertisements get assigned to which search terms, and how much the advertiser has to pay the search engine – are solved via keyword auctions. Advertisers choose keywords so as their ads will be displayed in answer to a user's query. In accordance with their own private assessment, each player submits a bid with indication of the maximum cost per click.

Before we give the necessary definitions, we should say a few words about the history of Internet auctions to facilitate the understanding of the problem. In the history of Internet advertisements, there are three main periods. In the first of them, since 1994, advertisers largerly paid a set price to display their ads a fixed number of times, basically, 1000 showing. Then in 1997 the Overture company (now, it is a part of Yahoo!) entered an entirely modern model of online auction – Generalized First-Price Auctions (GFP). In GFP each bidder submits a bid he is willing to pay per click for particular keyword. This model is significantly differ for advertisers: instead of showing their advertisements to everyone who are on the site, players can define a range of relevant keywords, so advertisements can immediately get to the potential buyer. However, it soon became clear that this mechanism was unstable and generally it was far from perfect. Being aware of all disadvantages of this system, Google offered its own pay-per-click system – AdWords Select. They learned that the bidder who is on position i will never be willing to pay more than the bid of the advertiser in position $(i + 1)$ plus a bid increment. Google applied this principle to GFP and, in February 2002, introduced a generalized second price auction mechanism (GSP). Interestingly, as early as in 2005 the annual GSP auctions revenue was billions of dollars.

Generalized Second-Price Auction is defined by:

- A set of n players with their own valuation v_i per click. Let us allocate it in decreasing order: $v_1 > \dots > v_n$. Each advertisers make a bid b_i in accordance with v_i.
- A set of s objects or slots - positions, which may contain advertising. Each slot has the probability α_i (click-through rates or CTRs) of being clicked by user: $\alpha_1 > \dots > \alpha_s$.

The mechanism of GSP:

- For each slot i the payments of winning player W_i is b_{Wi+1}.
- The utility function u_i of the player winning slot i is equal to $\alpha_i\left(v_i - b_{W_{i+1}}\right)$.

We consider a repeated keyword auction, with a fixed set of n players and s slots. The participants in such auction have the opportunity to update their bids for participation in between successive rounds. However, without any real idea of the competitive strategies followed by the other players, it is difficult for one player to make predictions about the future bids of other player. Thus, a natural approach is to assume that all the other bids will remain fixed in the next round. This leads to the following definition.

Definition 1. A greedy bidding strategy for each player is to choose for the next round a bid maximizing his utility u_i, assuming that the other players will repeat their bids. Suppose that s^* is a target slot to any of advertisers and p_{s^*} is a price for it. If player k is greedy, the allowable range of his bids is limited by the price per slot s^*_k and the price per slot with one step highter CTR. So, his class of strategies is defined as $b^*_k \in (p_{s^*}\,;\, p_{s^*-1})$.

Generally, there are two objectives pursued by the players participating in a keyword auction: the primary goal is to reach a target slot, the secondary objective is to make your competitors in auction to pay a higher price. A bidding strategy is mainly to balance these objectives. In accordance with this aim, if player uses the BB strategy, the utility functions for each bids from the class of strategies mentioned earlier must be equal: in a balanced bidding strategy player k chooses his bid b^* for the next round so as to satisfy the equation:

$$\alpha_{s^*_k}(v_k - p_{k^*_s}) = \alpha_{s^*_{k-1}}(v_k - b^*).$$

Theorem 1. [2, 242–259] In the BB auction with distinct CTRs, there is a unique fixed point, called a continuum of Nash equilibria. In this point the revenue of engine system is identical to those that would be gained in the VCG mechanism. The bids b^* in the equilibria follow equations:

$$b^*_k = \begin{cases} 2b_2, & k = 1; \\ v_k - \dfrac{\alpha_k}{\alpha_{k-1}}(v_k - b^*_{k+1}), & 2 \le k \le m; \\ v_k, & k \ge m+1. \end{cases}$$

We define two important models commonly used in practice. If all players simultaneously update their bids according to the BB strategy each round, we call such a game a synchronous model. And, conversely, in the asynchronous model the only one randomly chosen player can update his bids, while the other will continue to submit close to previous values.

Theorem 2. [3, 8-56] For a repeated keyword auction in which all players are following the BB strategy we have:

- For two slots, the BB auction always converges to a unique fixed point in both the synchronous and asynchronous models. In the synchronous model it takes place until a number of rounds is equal to $O\left(\log\left(\frac{v_2 - v_3}{v_3}\right)\right)$, where O depends on α_2

  the values of α_1 and .
- For three or more slots, BB does not necessarily always converge in the synchronous case.
- In the asynchronous model where players bid in random order, no matter how many slots there are, the BB strategy always converges. But it does not hold, if the bidders are not chosen randomly. Along with this, convergence occurs on average in $O(r_1(n \log k) + \log n + k^{r_2})$ steps, where:

  (a) $r_1 = 2 + \log_{t^*}(1 - t^*)\frac{v_k - v_{k+1}}{v_{k+1}}$, where $t^* = \max_{k \geq 0} \alpha_k / \alpha_{k-1}$;

  (b) $x = \log_{1/t^*} \frac{v_1 - v_{k+1}}{\varepsilon}$;

  (c) $r_2 = 2^k(1 + x)$.

Next theorem shows how much income the GSP mechanism brings to search engines and compares this result with auctioneer revenue in the case where all players are using the VCG mechanism.

Theorem 3. [1, 262–271]

- There exists a GSP keyword auction with a Nash equilibrium whose revenue is at most times the revenue of the VCG mechanism for every $K > 0$. Moreover, the bid of each bidder is less then their private valuation $b_i \leq v_i$.
- If a valuation $(b_i \leq v_i)$ is satisfied, then for every GSP keyword auction with a

Nash equilibrium whose revenue is at most times the revenue of the VCG mechanism.

- There exists a GSP keyword auction with a Nash equilibrium whose revenue is at least K times the revenue of the VCG mechanism for every $K > 0$.

In this work, we have briefly described two commonly used auction systems and investigated their most useful properties. In the course of the solution of the task, we have considered a natural class of bidding strategies, seeking to maximize the advertiser’s utility function, provided that the behaviour, of the other participants is under limited assumptions. Our analysis has shown how the advertisers who are using a greedy strategy can reach the most natural of the GSP equilibrium. Finally, we have determined the expected amount of rounds the bids converge.

References

1. Cary, M., Das, A., Edelman, B., Giotis, I., Heimerl, K., Karlin, A. R., Mathieu, C. & Schwarz M. (2007). Greedy Bidding Strategies for Keyword Auctions. In Eighth ACM Conference on Electronic Commerce, 242–259;

2. Edelman, B., Ostrovsky, M. & Schwarz M. (2006). Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords. American Economic Review, 242–259;

3. Cary, M., Das, A., Edelman, B., Giotis, I., Heimerl, K., Karlin, A. R., Mathieu, C. & Schwarz M. (2008). On Best-Response Bidding in GSP Auctions. Harvard Business School Working Papers, 8-56.